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Study on the Structural Behavior and Design of a Typical Single Cell Post Tensioned Concrete Box Girder Bridge

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Abstract : *In present work, a single cell post tensioned concrete box girder with simply supported ends has been analyzed using finite element method. SAP2000 software is used to carryout linear analysis using 4-noded thin shell element. Pre-stressing force and losses have been included in the modelling. A comparative study carried on longitudinal and transverse bending stress, shear stress, torsional shear stress both by SAP2000 software and manual calculation. Simple beam theory is adopted for calculation of longitudinal flexural stress and shear stress across the section. Beam on elastic foundation analogy is used for analysis of transverse bending stress due to asymmetrical loading. A typical 40m rectangular concrete box girder bridge is considered in analysis and design. The load cases such as dead load, superimposed dead load and live load as per IRC specifications Class A-one lane, Class A-two lane and IRC 70R-one lane loading are considered in analysis and design. Box girder is designed by post tension method with straight and parabolic tendons, including prestress losses, checked for permissible stresses as per IRC: 18-2000. The percentage difference of results obtained between manual calculations and software for various types of stresses patterns are plotted, discussed and results are tabulated.*

Keywords: Concrete Box Girder; Bending Stress; Shear Stress; Post Tension Design; Prestress Loss; SAP2000

1. INTRODUCTION

Box girders, have gained wide acceptance in freeway and bridge systems due to their structural efficiency, better stability, serviceability, economy of construction and pleasing aesthetics.

Long span girder bridges with more than 25m, with wider deck, suffers longitudinal and transverse distortion of cross section under eccentric load. Hence such girder bridges require high torsional rigidity to keep the effect of distortion of the deck to be minimum. As span increases, dead load increases. To reduce the dead load, unnecessary material near the center of gravity. A box girder is formed when two web plates are joined by a common flange at both the top and the bottom. The box girder normally comprises either prestressed concrete, structural steel, or a composite of steel and reinforced concrete. It typically rectangular or trapezoidal in cross section. Nowadays these box girder are used in flyovers, elevated metro bridges, casted by segmental construction or integral one.

1.1 Advantage of box girder

- In case of long span bridges, large width of deck is available to accommodate prestressing cables at bottom flange level. The maintenance of box girder is easier in interior space is directly accessible without use of scaffolding.
- It has high structural efficiency which minimizes the prestressing force required to resist a given bending moment, and its great torsional strength with the capacity this gives to re-center eccentric live loads, minimizing the prestress required to carry them
- They could be cast in smaller segments and could be integrated into one unit by prestressing to achieve longer span

The box girder often is more advantageous than t-beam due to its high bending stiffness combined with a low dead load, yielding a favorable ratio of dead load to live load. Its high torsional stiffness which allows freedom in the selection of both the supports and bridge alignment,

Analysis and design of box-girder bridges are very complex because of its three dimensional behaviors consisting of torsion, distortion and bending in longitudinal and transverse directions.

2. STRUCTURAL BEHAVIOR

Force coming on the deck leads to longitudinal bending, transverse bending and interaction of longitudinal and transverse bending stresses. Consider an example, live load 2Q acting on box girder due to vehicular movement as shown in below figure 2.1. This load can be resolved into equivalent symmetrical vertical forces P_1, P_2 and moment M at ends of webs. The effect of moment at this stage cause only local transverse flexure.

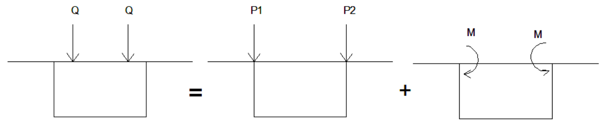

Figure 2.1 Resolution of force system

Consider the symmetrical vertical forces acts at webs are denoted by P_1 and P_2 in figure 2.2. As shown in below figure, (a) = (b) +(c). Since (c) = (d) + (e), it is evident that (a) = (b) + (d) + (e). Now (b) and (d) are symmetrical loads and, as in the case of superimposed dead loads and self-weight, do not create any torsional effects. These symmetrical loads cause simple longitudinal flexure only. The asymmetrical loads, as shown in (e) from below figure, cause torsional effects, which in turn cause rigid body rotation (pure torsion) and distortion of section.

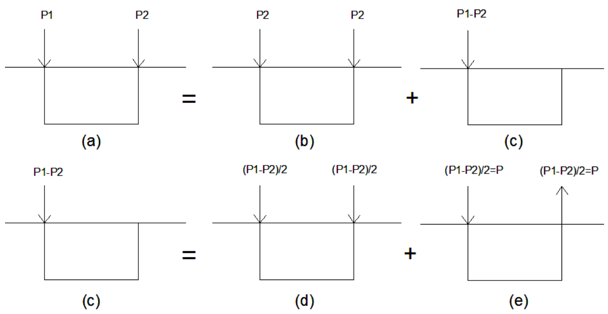

Figure 2.2 Different reactions due to live load on the deck

Structural action while resisting the external loads are,

1. Simple beam action in the longitudinal direction causing longitudinal flexural stresses and shear stress across the section.
2. Torsion of the cross section due to eccentricity of loading which involves St. Venant's shear stress and warping stresses in the longitudinal direction.
3. Distortion of the section due to eccentric loading which causes transverse bending stress, shear stress across the section, longitudinal warping stresses (distortional) and corresponding warping shear stresses.

2.1 Longitudinal Flexural stress and shear stress

Simple beam theory is adopted for calculation of longitudinal flexural stress and shear stress across the section. If M_x and M_y are bending moments, act on the section, the normal stress in longitudinal bending of thin walled beam whose cross section has vertical axis of symmetry is given by,

$$f_{lbg} = \frac{M_x y}{I_x} + \frac{M_y x}{I_y} \quad \text{… (Equation 2.1)}$$

Where,

f_{lbg} = normal longitudinal stress in bending (positive tensile)

I_{xx}, I_{yy} = moment of inertia of the entire cross section about centroidal x and y axis

x, y = coordinates of the point in the middle of the cross section.

For the shear stress arising in longitudinal bending due to vertical loading only, by symmetry about vertical axis of cross section, the longitudinal shear stress is zero at this axis. Hence the complimentary shear stress V_{lbg} in the plane of cross section is also zero

Shear in longitudinal bending is given by,

$$V_{lbg}(h) = -\frac{V_y (Ay)}{I_x} \quad \text{… (Equation 2.1.1)}$$

Where,

V_{lbg} h = Shear flow in longitudinal bending

V_{lbg} = Shear stress in longitudinal bending

h = thickness of wall

V_y = shear force on the cross section in Y-direction

(Ay) = First moment of area of the partial half of cross section about centroidal x axis.

2.2 Torsional stresses

St.Venant stresses which can be evaluated by the elemental theory of torsion as applied to closed sections of thin walled members.

St. Venant torsional shear stress V_{svt} due to live load

$$V_{svt} h = \frac{T_{svt}}{2A_{enc}} \quad \text{… (Equation 2.2.1)}$$

V_{svt} h = shear flow in St. Venant torsion

V_{svt} = shear stress in St. Venant torsion

h = thickness of wall

T_{svt} = torsional moment applied on a section

A_{enc} = bd, area enclosed by mid-line of wall of enclosed.

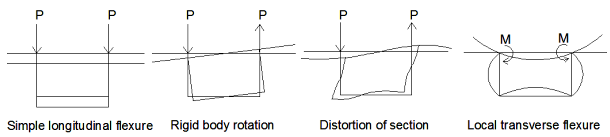

Figure. 2.2 Overall structural behavior of box girder

3. BEAM ON ELASTIC FOUNDATION ANALOGY

Box girder subjected to unsymmetrical transverse loading undergoes deformation leading to distortion of the section, giving rise to transverse stresses as well as longitudinal warping stresses.

Beam on elastic foundation is adopted because

1. Support given by adjacent cross section to resist transverse bending deformation is elastic and this reduces to zero with a distance. Hence the behavior of the box cell due to distortion is analogous to a beam on elastic foundation.
2. The deformation of top and bottom corners of the web is elastically controlled by flexure stiffness top and bottom slabs. This also shows that the distortional behavior of box section due to eccentric loading leading to distortion and transverse bending is analogous to the behavior of beam on elastic foundation.

3.1 Analysis transverse bending stress

The line diagram of the box girder is shown below. To evaluate warping deflection *w*, of the loaded nodes of cell, consider deflection denoted by δ, for the unit uniform torsional load. It is computed by unit length of cell, made statistically determinate by cutting the bottom plate at its midpoint. The out of plane shear in bottom flange per unit torsional load is

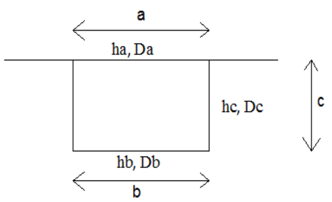

Figure 3.1 Notations used for analysis

$$v = \frac{\frac{1}{D_c}[(2a+b)\,abc] + \frac{1}{D_a}(ba^3)}{(a+b)\left[\frac{a^3}{D_a} + \frac{2c(a^2+ba+b^2}{D_c} + \frac{b^3}{D_a}\right]}$$ ….. (Equation 3.1)

The vertical defection of one web per unit torsional load is

$$\delta = \frac{ab}{24(a+b)}\left\{\frac{c}{D_c}\left(\frac{2ab}{(a+b)} - v[2a+b]\right) + \frac{a^2}{D_a}\left(\frac{b}{a+b} - v\right)\right\}$$

…. (Equation 3.2)

Where,

a = distance between c/c of web

b = distance between c/c of web

c = distance between center of deck slab and soffit slab

h_a, h_b, h_c = thickness of deck slab, soffit slab, web respectively

D_a = flexural rigidity of deck slab $(1*h_a^3/12)$ E

D_b = flexural rigidity of soffit slab $(1*h_b^3/12)$ E

D_c = flexural rigidity of web $(1*h_c^3/12)$ E

The spring constant required for analysis of beam on beam on elastic foundation is calculated as

$K = 1/\delta$

Transverse bending moment Mt due to asymmetrical loading can be evaluated as, $M_t = F_d\ y\ k\ S$

y = deflection of beam on elastic foundation

K = foundation modulus

S = section modulus = $(b*h^2/6)$

F_d = top flange and web = $F_d = \frac{b_{top}}{2S}\left[\frac{b_{top}}{(b_{top}+b_{top})}\right]$

F_d = bottom flange and web = $F_d = \frac{b.v}{2S}$

4. GEOMETRY, MODELING AND MATERIAL PROPERTIES

The span to depth ratio can be around 17 to 18 for reinforced concrete section and 21 to 25 for prestressed concrete. Preliminary cross section dimensions are calculated from IRC 18:2000, clause 9.3.2 as shown in figure 4.1. Haunches of minimum size of 300mm (horizontal) and 150 mm (vertical) provided at bottom inner corners and 1000mm (horizontal) and 150 mm (vertical) at top inner corners of the box section.

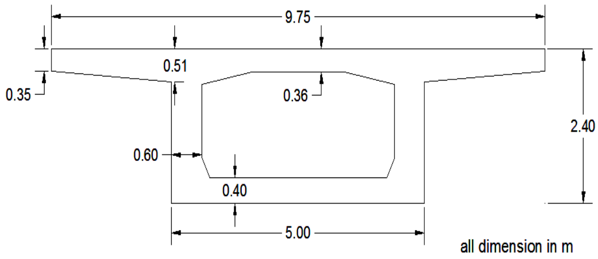

Figure 4.1 Cross section of box girder

Table 4.1 Cross section properties of box girder

Span	40 m	A	8.5 m^2	Y_t	1.06 m	Z_t	6.13 m^3
width	9.75 m	I_{xx}	6.5 m^4	Y_b	1.34 m	Z_b	4.85 m^3

Table 4.2 Properties of Material

Properties	Value
Concrete (grade M40)	
Characteristic strength, fck	40 Mpa
Young's modulus, Ec	5000√40 Mpa
Density (normal weight concrete)	25 kN/m^3
Steel reinforcing bar (Fe 415)	
Yield stress, fy	415 MPa
Ultimate tensile strength, fu	485 MPa

4.1 Finite Element Modeling

The box girder model has been analysed by area object using Bridge Modeller of SAP2000. The software program uses the matrix displacement method of analysis based on finite element idealization. The entire section of the box has been modelled Using 4-noded thin shell elements. The shell element has both bending and membrane capabilities. Both in-plane and normal loads are permitted. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes.

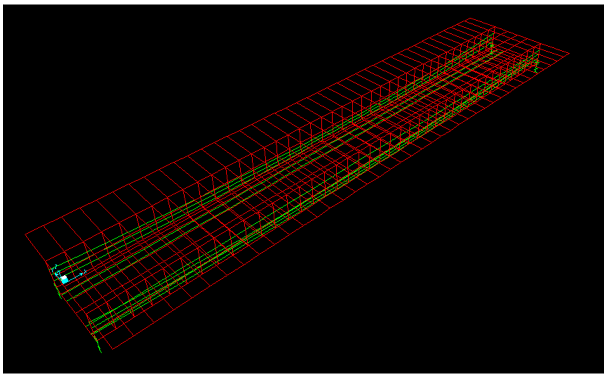

Figure 5.1. Finite Element model of Box girder with layout of cables created in SAP 2000

To simulate the effect of prestressing, tendons are modeled as elements by inputting its cross section area and prestress load. Using tendon quick start option, straight and parabolic tendons are modeled with calculated eccentricity from center of gravity of concrete. Finite Element model of Box girder created in SAP 2000 as shown in figure 5.1.

5. ANALYSIS OF BOX SECTION

5.1 loads

The load cases considered for two lane traffic such as dead load, superimposed dead load and live load as per IRC: 6-2014 specifications are considered in analysis and design

Dead Load: Based on cross section area, for density 25 kN/m^3 self-weight is calculated.

Superimposed Dead Load: 40 kN/meter run. It includes crash barrier, wearing coat and service loads

Live Load: Includes impact factor to obtain maximum bending moment and shear force

- IRC Class A- one lane load
- IRC Class A- two lane load
- IRC 70R- one lane load

5.2 Longitudinal Flexural stress and shear stress

Calculation of dead and live moment

Total dead load = 212.5 kN/m

Total live load concentrated at midspan over the web = 1200 kN

Midspan bending moment = (212.5*40^2/8) + (1200*40/4)

M_x = 54500 kNm

Shear force at end, V_y = (8500+1200)/2 = 4850 kN

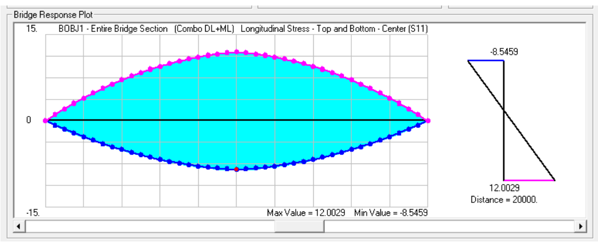

Figure 5.2 Longitudinal Bending from SAP at midspan

Table 5.2 Longitudinal Bending stress at mid-line

	Calculated stress	Stress from SAP
Top slab, f_{lbg} N/mm^2	-8.8	-8.54
Bottom slab, f_{lbg} N/mm^2	11.23	12

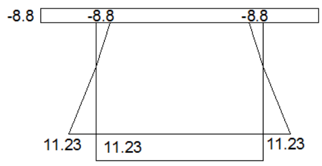

Figure 5.2 Longitudinal Bending stress (f_{lbg}) pattern at midspan from beam bending theory

Shear stress from Equation 2.1.1 are calculated for the sections a, b, c, d and e chosen as shown in below Figure 5.3 with shear stress pattern. Due to space limitation stress results only from manual calculation are presented from here onwards.

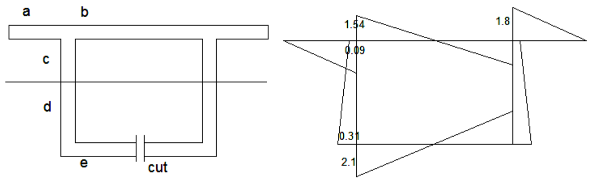

Figure 5.3 Shear stress pattern with sections

St. Venant torsional shear stress V_{svt} due to live load from equation 2.2.1 is calculated as follows

Shear stress in St. Venant torsion, V_{svt} = 1395 kNm

area enclosed by mid-line of wall, A_{enc} = 12*10^6 mm^2

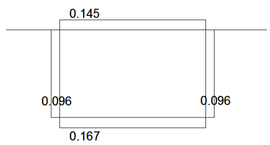

Figure 5.4 St. Venant torsional Shear stress pattern

Transverse bending moment M_t due to asymmetrical loading can be evaluated as, $M_t = F_d\ y\ k\ S$ based on beam on elastic foundation concept explained in section 3.2.

E = 5000√40 = 31622.776 N/mm^2 = 3162277.66 t/m^2

y = 0.34 mm, (from software output), k = 23860 MN/m^2

Table 5.2.1 Dimensions for analysis

length, m		thickness, m		flexural rigidity(EI), t/m^2	for 1m length
a	4.71	Ha	0.36	Da	12294.936
b	4.71	Hb	0.4	Db	16865.481
c	2.134	Hc	0.6	Dc	56920.998

Table 5.2.2 Values of transverse moment

F_d	flange	web	transverse moment M_t t-m	flange	web
top	54.5	19.63	top	9.59	9.6
bot	11.7	26.23	bot	12.7	12.7

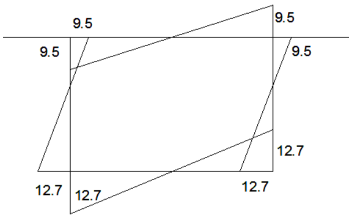

Figure 5.5 Transverse moment diagram

6. DESIGN STEPS IN POST TENSION METHOD

Prestress sections under the action of flexure should satisfy the limit specified for permissible stresses at the stage of transfer of prestress and at service loads. For class 1 type of structure no tensile stress at transfer and service.

$f_{ci} = 0.8*40 = 32$ N/mm^2, loss ratio, $\eta = 0.8$,

$f_{ct} = 0.5*f_{ci} = 16$ N/mm^2 (Fig. 8 from IS 1343:2012)

$f_{cw} = 0.39*f_{ck} = 15.6$ N/mm^2 (Fig.7 from IS 1343:2012, for zone I)

f_{ct}, f_{cw} = allowable compressive stress in concrete at initial transfer of prestress and under service loads respectively

f_{tt}, f_{tw} = allowable tensile stress in concrete at initial transfer of prestress and under service loads respectively

Range of prestress, $f_{br} = 12.8$ N/mm^2, $f_{tr} = 15.6$ N/mm^2

Check for minimum section modulus

$$Z_t \geq \left[\frac{(M_q + M_g) - \eta M_{min}}{f_{tr}}\right], \quad Z_b \geq \left[\frac{M_d - \eta M_{min}}{f_{tr}}\right]$$

Table6.1 dead load and live load moment

DL BM	Mg=Mmim	42500, kN/m
LL BM	Mq	4500, kN/m
Md	(Mg+Mq)	47500, kN/m

Range of prestress developed in concrete at top and bottom, $f_{sup} = -6.9$ N/mm^2, $f_{ind} = 12.23$ N/mm^2,

Minimum prestress force, P = 13634 kN

$$P = \left(\frac{A(f_{inf} Z_b + f_{sup} Z_t)}{(Z_t + Z_b)}\right)$$

Limiting zone for the prestress force

$$e \leq \left[\frac{-Z_b f_{ct}}{P} - \frac{Z_b}{A} + \frac{M_{min}}{P}\right],$$

$$e \geq \left[\frac{-Z_b f_{tw}}{\eta P} - \frac{Z_b}{A} + \frac{M_d}{\eta P}\right]$$

The theoretical value of the eccentricity determined from the above expression is impracticable since it falls at bottom edge of section. Hence by increasing prestress force to 70000kN, a new zone of limiting eccentricity for cables is obtained.

Table 6.2 Limiting zone of ecentricty

	midspan, mm	end span, mm
transfer of prestress, e_b	1139.57	532.43
service loads, e_t	274.57	-564.7

Using Freyssinet system, 6 number of 12K15 straight cables and 4 number of 19K15 are provided as per IS: 6006-2008 to counteract load on box girder. Eccentricity of all cables at support is 301.2mm and midspan is 860mm from center of gravity of concrete.

Table 6.1 ecentricity from bottom

cable no.	endspan	midspan	cable no.	endspan	midspan
1	200	200	7	1200	500
2	200	200	8	1200	500
3	200	200	9	1600	750
4	200	200	10	1600	750
5	750	250			
6	750	250			

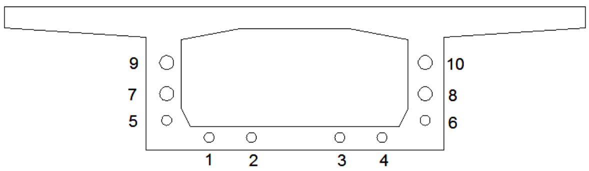

Figure 6.1 Location and numbering of cables

6.1 Calculation of prestress losses

Different types of losses of prestress as per IS 1343 are presented below

Total elongation loss = 16.92 32 N/mm^2

Shrinkage of concrete = 26.4 N/mm^2

Creep of concrete = 167 N/mm^2

Relaxation of stress in steel = 95 N/mm^2

Anchorage slip of each cable = 24.3 N/mm^2

Loss due to friction = 118 N/mm^2 (cable no. 1, 2, 3, 4), 161.8 N/mm^2 (cable no. 5, 6), 179 N/mm^2 (cable no.7, 8) 191.9 N/mm^2 (cable no. 9, 10)

6.2 Calculation of stresses at top and bottom

Permissible temporary stresses in concrete, permissible stress in concrete during service limits are given in IRC: 18-2000, clause 7.

Tensile stress at transfer = 3.12 < 4 N/mm^2. No tensile stress during service.

Compressive stress at transfer = 6.5 <$0.5*f_{cj}$ = 20 N/mm^2. During service = 7.39 <$0.33*f_{ck}$ = 13.2 N/mm^2

Final stresses are within specified limits.

Table 6.2.1 Calculation of stresses at top and bottom

at transfer	s_t	3.12 N/mm^2
	s_b	6.514 N/mm^2
at service load	s_t	0.0331 N/mm^2
	s_b	7.399N/mm^2

7. CONCLUSION

This paper gives basic principles for analysis of box girder by simple beam theory and beam on elastic foundation.

1. Percentage difference between results from simple beam theory and finite element method for longitudinal analysis is 2.95% for top slab and-6.85% for bottom slab.
2. Shear stresses obtained at the junction of webs and flanges are more compared with stresses in web portions.
3. Torsional shear stresses obtained is very less because, for box girder bridge dead load distributed uniformly which is much more than live load from vehicle. Box girder gives very high torsional rigidity.
4. St.Venant torsional shear stresses adopted is only for thin walled members of closed sections.
5. Distortional warping stress is caused by variation in the transverse bending curvature along the length is 20 percent of longitudinal bending stress due to beam bending.
6. Inclined webs of box girder behaves structurally better based on force flow condition. Trapezoidal box girder offers more resistance to shear generated.
7. The effect of distortion of cross section can be restricted by providing diaphragms at regular interval, which improve bending stiffness of web and flange.

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